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13. ABSTRACT *(Maximum 200 Words)*
The general goal of this research is to develop the modern mathematical theory of nonlinear dispersive waves, and to apply this theory to applications in nonlinear optics. Aspects of the work also study fundamental properties of nonlinear materials, such as liquid crystals and polymers. These foundational studies provide basic understanding of nonlinear processes which are important for technological applications relevant to the Air Force, such as laser hardening for protection against intense optical pulses. Primary results of these studies include: (i) A description of propagation effects in highly nonlinear liquid crystal media; (ii) The characterization of properties of random, chaotic,and turbulent nonlinear waves; (iii) A collaboration between members of Brooks AFB, Wright Patterson AFB, and the Courant Institute to investigate laser hardening via optical limiting with reverse saturable absorption; (iv) A study of transverse effects in all-optical bistable arrays; (v) A study of precursors in idealized nonlinear propagation; (vi) A study of the dynamics of polymer and polymer/liq crystal systems; and (vii) the resolution through nonlinearity of singular interactions between reflected and diffracted waves.

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1 Executive Summary

The objective of this research is to develop the modern mathematical theory of nonlinear dispersive waves, focusing on areas relevant for nonlinear optics, and simultaneously to apply this theory to problems in nonlinear optics which are relevant to the missions of the Air Force. In addition, some aspects of the work concern the properties and behaviors of nonlinear materials, such as liquid crystals and polymers. While fundamental theoretical and mathematical behavior is the focus of these studies, it must be emphasized that such basic understanding is essential to applications of direct importance to the Air Force - such as laser hardening to protect against intense optical pulses of both short and long duration.

This grant provided support for the PI (Professor McLaughlin), two senior colleagues (Professors E and Ting), and several post-docs and graduate students. Yuchi Chen completed his PhD under this grant. The support for A. Paul and S. Kim, post docs at Brooks AFB under the supervision of Dr. Mary Potasek, should be particularly noted. In addition, the grant also provided funds for senior visitors Philip Rosenau and Thomas Ernaux. Finally, McLaughlin's research benefited from continual collaborations with Professors Michael Shelley and Jalal Shatah, who did not receive explicit support from the grant.

The primary research results which were obtained during the time period of this grant are briefly described below. Most have appeared in the published literature or in a manuscript submitted for publication, and are listed in the "Publications" below. These research results include:

(i) McLaughlin (with Muraki and Shelley) completed the first coupled study of laser propagation in nematic liquid crystals. [This group formed the theoretical component of a larger effort which included the physical experiments of Braun, Faucheux, and Libchaber at Princeton University.] The extreme nonlinearities of nematic liquid crystals produce striking and unusual behavior such as filamentation and undulation of the laser beam as it propagates through the liquid crystal. These new observed phenomena were understood qualitatively through asymptotic and numerical modeling.

(ii) McLaughlin has established and created a major research program to identify and understand random, chaotic, and turbulent behavior in nonlinear dispersive wave systems. Many parts of this program were completed during this three years of AFOSR support, including: **(a)** The completion (with Majda and Tabak) of an initial study of the weak turbulence theory for a model one dimensional dispersive wave system. This study, which has had considerable impact on the nonlinear wave community, demonstrates that weak turbulence theory is not valid for this system. More importantly, it identifies one source of the invalidity, and proposes a new alternative closure which fits the numerical data extremely well. **(b)** The establishment (with Jalal Shatah) of the existence of homoclinic orbits with complex spatial structure for perturbed NLS equations. This existence result (which is the first of its kind for pde's) provides the backbone for a temporal chaos of nonlinear dispersive waves which involves the jumping of a localized spatial wave between distinct spatial locations. As such, this type of chaos is easy to observe in deterministic wave systems. Haller (an ex post-doc at Courant Institute) has been able to use this backbone to establish the existence of complex transients, and Li (an ex graduate student of McLaughlin) has been able to use similar homoclinic structures in finite dimensional systems to define a model symbol dynamics, with the distinct spatial locations as its symbols. **(c)** The initiation (with Cai and Shatah) of a study of the simultaneous presence of space-time chaos in larger, spatially extended domains. **(d)** The completion (with Bronski and Shelley) of a theoretical and numerical study of nonlinear Schroedinger (NLS) propagation through a one-dimensional random media. This study identified distinct behaviors of random waves in focusing and defocusing media, both of which are very different from each other, as well as from the striking localization of the linear case.

(iii) McLaughlin [with Mary Potasek (Brooks AFB) and Ruth Pachter (Wright Patterson AFB)] initiated a study of propagation in dissipation dominated media, tailored for application to laser hardening in the visible wavelength. The goal of this multi-laboratory (Brooks AFB, Wright Patterson AFB, and the Courant Institute) study is to use ab-initio quantum calculations of the media, measurements of the media response characteristics, and propagation codes for the investigation of light-matter interactions for high power, short pulses of laser light in the visible-infrared sub-picosecond range (50 femtoseconds to picoseconds).

(iv) McLaughlin (with Y. Chen) completed a study of all-optical bistable arrays, in the presence of propagation, diffraction, and strong diffusion. Their primary result is that a significant enhancement of the performance of the array is possible with focusing nonlinearities, even with a very small amount of diffraction.

(v) McLaughlin (with T. Ueda) have in progress a study of precursors for model nonlinear wave equations. This study focuses upon the energy content of the precursors – comparing and contrasting precursors in nonlinear systems with those in linear systems. The study is an outgrowth of a study of precursors for storm tracks in the atmosphere, carried out by Majda, McLaughlin, Overman, and Ueda.

(vi) Professor E (with Palffy-Muhoray and Otto) have completed a thorough and systematic study of the dynamic behavior of polymer systems, focusing upon a visco-elastic description of flexible polymers. Currently, E (with Muratov) is extending this work to rigid polymer systems such as polymer/liquid crystal systems.

(vii) Professor E has also studied light propagation in general homogeneous media (with Palffy-Muhoray and Yuan), and is developing (with Palffy-Muhoray) a theory of orientational ratchets in the photoalignment of liquid crystals.

(vii) Ting (with Keller) has resolved the classical problem of a singularity near a line of tangency of an incident or reflected wave with a diffracted wave, first posed by Lighthill. This defect of linear wave theory was resolved by introducing a canonical nonlinear elliptic problem which eliminated the singularity. The method enabled them to resolve singularities in self-similar flow fields behind plane shocks diffracted by wedges and corners, and also in the steady flows past thin wings with supersonic leading edges.

(viii) Ting also continued his investigations of vortex filaments, their interaction with solid boundaries, and sound generation.

2 Technical Report

Laser Propagation in Nematic Liquid Crystals: Nematic liquid crystals are media with extremely strong nonlinearities. As such, they provide candidate materials for applications of direct interest to the Air Force such as "laser hardening". The nonlinearity in these materials is so strong that it produces new and unexpected effects when laser beams propagate through such media – effects which require both experimental and theoretical explanation. This work provides one of the first experimental-theoretical studies of such propagational effects in nematic liquid crystals.

In recent laboratory experiments which investigate the nonlinear interaction between light and nematic liquid crystals, Braun, Faucheux and Libchaber observed complex optical beam structures that were generated by the strong self-focusing of laser light. A simplified partial differential equations (pde) model is developed, which captures the essential coupling between optical refraction and nematic deformation and demonstrates that two of the experimentally observed features – *undulation* and *filamentation* – are direct consequences of the generation of spatial scales by strong nonlinearity.

For the mathematical analysis, a novel new asymptotic representation is developed for this strongly coupled nonlinear system which exploits the natural separation of scales at which these optical structures are created by the self-focusing process. This representation combines geometrical optics, paraxial optics, and free boundary asymptotics to produce a closed, "outer free boundary problem" which describes the undulation of the field, and an "inner" nonlocal - nonlinear Schroedinger equation which describes the filamentation structure of the light wave on the inner scale. The outer free boundary problem is studied numerically with a "boundary integral" method, while the nonlocal NLS equation is solved with an "integrating factor method". Thus, the pde model is reduced asymptotically into two decoupled systems at two distinct spatial scales, each of which is studied numerically to produce a theoretical corroboration of the unusual nonlinear optical behaviors of "undulation" and "filamentation".

This theoretical work of McLaughlin, Muraki and Shelley was then extended (with Wang) through numerical computations which represent a direct emulation of the experimental configuration within the context of a paraxial pde model. In addition to providing numerical corroboration of the earlier asymptotic analysis, these numerical experiments suggest that focusing caustics play a crucial role in the formation of beam filament pairs in this strongly nonlinear system.

Weak Turbulence Theory: Nonlinear random waves, where the randomness originates from initial conditions, and when the waves are small amplitude and hence weakly nonlinear, are often described by "weak turbulence theory". This theory, while based on beautiful physical intuition, is also based on uncontrolled and heuristic approximations which have not been verified by careful numerical experiments. Majda, McLaughlin and Tabak have designed a simple one dimensional model problem with which weak turbulence could be tested numerically.

A family of one-dimensional nonlinear dispersive wave equations is introduced as a model for assessing the validity of weak turbulence theory for random waves in an unambiguous and transparent fashion. These models have an explicitly solvable weak turbulence theory which is developed, with Kolmogorov-type wave number spectra exhibiting interesting dependence on parameters in the equations. These predictions of weak turbulence theory are compared with numerical solutions with damping and driving that exhibit a statistical inertial scaling range over as much as two decades in wave number.

It is established that the quasi Gaussian random phase hypothesis of weak turbulence theory is an excellent approximation in the numerical statistical steady state. Nevertheless, the predictions of weak turbulence theory fail and yield a much flatter ($|k|^{-1/3}$) spectrum compared with the steeper ($|k|^{-3/4}$) spectrum observed in the numerical statistical steady state. The reasons for the failure of weak turbulence theory in this context are elucidated. Finally, an inertial range closure and scaling theory is developed which successfully predicts the inertial range exponents observed in the numerical statistical steady states.

Homoclinic Orbits for Pde's: Some integrable nonlinear wave equations (such as the focusing NLS equation of nonlinear optics) possess periodic and quasiperiodic solutions which are unstable ("unstable tori"). [McLaughlin (with Ercolani and Forest) were the first to obtain representations of the resulting homoclinic orbits, under earlier AFOSR support.] When such integrable wave equations are weakly perturbed, numerical experiments establish that these instabilities produce solutions which behave irregularly in time - temporal chaos in deterministic nonlinear wave systems. McLaughlin's goal has been to understand precisely, and mathematically, temporal chaos in such near-integrable systems. A crucial analytical step in this process is to establish the persistence of the homoclinic orbits under perturbations, for these persistent homoclinic orbits provide a backbone which supports chaotic behavior.

McLaughlin (with Li, Shatah, and Wiggins) has obtained the first persistence result for pde's, when the unperturbed orbit is nontrivial. More precisely, they have established the following:

Theorem: The perturbed NLS equation possesses a symmetric pair of orbits which are homoclinic to a saddle fixed point, provided the parameters lie on a codimension 1 set in parameter space which is approximately described by the line relating two coefficients of dissipation,

$$\alpha = E(\omega)\beta.$$

Formulas exist which describe, approximately, the characteristic properties of these homoclinic orbits such as the slope $E(\omega)$ and an important "take-off" angle.

This theorem is a major result in evolutionary pde's. It establishes the existence of a symmetric pair of homoclinic orbits for the perturbed pde - orbits which are singular deformations of orbits for the integrable case, and which possess spatial structures localized either near the center or the edge of the periodic box. Thus, this theorem is a key step toward a pde symbol dynamics on two symbols [C (center) and E (edge)] – that is, toward establishing the existence of motions for the pde as random as a sequence of "coin tosses". Moreover, the proof of this theorem introduces, develops, and/or uses new methods for the global analysis of evolutionary pde's – Melnikov analysis, geometric singular perturbation theory, normal forms, and high dimensional shooting methods. These developments follow McLaughlin's earlier work on finite dimensional discretizations of the pde. This earlier work, supported by previous Air Force Grants, was designed to develop methods for large finite dimensional systems which would extend to pde's. The above theorem established this extension.

However, these geometric Melnikov arguments are somewhat cumbersome. McLaughlin and Shatah replaced this analysis with a simpler Lyapanov Schmidt (nonlinear Fredholm Alternative) procedure. Specifically, the persistence of homoclinic orbits under small perturbations of the wave equations, of both dissipative and conservative type, has been established. An analytic perturbation method based on time dependent scattering theory, and Fredholm theory, is used to establish this persistence. The estimates are given in space-time function spaces, with a certain time decay, which is required for the existence of a homoclinic orbit.

Finally, an important remark: Usually existence results in mathematics provide no insight into the qualitative behavior of solutions. However, the persistence of homoclinic orbits is an exception to this rule. The long time behavior of these orbits is known, and it contains fascinating and complex space-time structure which forms the backbone for temporally chaotic behavior. Pde existence results carrying this much information are rare indeed.

Spatial-Temporal Chaos: McLaughlin (with Cai, Wielaard, and Shatah) initiated a study of chaotic behavior in nonlinear wave systems with large spatial domains. In this setting, spatial and temporal chaos are expected to coexist. This is important because such random behavior is characteristic of turbulent waves in nature. Given such coexistence of spatial and temporal chaos, the response of the system requires stochastic analysis, even though the system itself is entirely deterministic. Numerical studies for such spatially extended systems were begun. The goals are (i) to identify systems with temporally chaotic behavior and which simultaneously possess rapidly decaying spatial correlations, and (ii) to correlate such behavior with the presence of hyperbolic structures and unstable manifolds of very high dimension. Numerical results have been obtained for damped and driven perturbations of the nonlinear Schroedinger equation, and for the complex Ginzberg Landau equation in both the amplitude and phase chaos regimes. To date. the main discovery is that *mutual information* is a better measure of the loss of spatial correlation than the frequently used two-point correlation functions.

Nonlinear Localization: McLaughlin (with Bronski and Shelley) have completed their study of scattering by a nonlinear, random slab. Propagation by a one-dimensional linear Schroedinger equation is totally destroyed by the addition of *any* amount of random potential, spread throughout the infinite domain. This striking and well known linear phenomena is rather completely understood – in contrast to propagation through a nonlinear random media, about which almost nothing is known mathematically.

Bronski, et.al., have studied (numerically and analytically) the problem of localization in a disordered one-dimensional nonlinear medium modeled by the nonlinear Schroedinger equation. Previous authors have shown that almost every time-harmonic solution of this random PDE exhibits localization. Bronski, et al, consider the temporal stability of such time-harmonic solutions and derive bounds on the location of any unstable eigenvalues. By direct numerical determination of the eigenvalues they show that these time-harmonic solutions are typically unstable, and find the distribution of eigenvalues in the complex plane. The distributions are distinctly different for focusing and defocusing nonlinearities. They argue further that these instabilities are connected with resonances in a Schroedinger problem, and interpret the earlier numerical simulations in terms of these instabilities. Finally in the defocusing case they are able to construct a family of asymptotic solutions which includes the stable limiting time-harmonic state observed in the simulations of Shelley.

Laser Hardening: McLaughlin [with Mary Potasek (Brooks AFB) and Ruth Pachter (Wright Patterson AFB)] have initiated a project for the investigation of light-matter interactions for high power, short pulses of laser light in the visible-infrared sub-picosecond range (50 femtoseconds to picoseconds). The goal is to focus upon adequate microscopic descriptions and measurements of the medium, and the interaction of this microscopic behavior with a macroscopic description of electromagnetic pulse propagation. It should be emphasized that both aspects of this project, the macroscopic propagation and the microscopic determination of material properties, are essential in many realistic applications of specific interest to the Air Force such as "laser hardening", protection of electronic circuits from laser damage, and the design of optical limiters. In such applications, reflection and propagation properties of macroscopic electromagnetic fields are fundamental; yet, the optimal design of materials for a specific application requires microscopic modeling. Neither can be carried out without the other. This project will involve a direct collaboration between members of three research groups – from the Courant Institute, Brooks Air Force Base, and Wright Patterson Air Force Base.

In this short pulse regime (in contrast with the extreme ultrashort regime), the electric field has many optical cycles, and an envelope description of the electromagnetic pulse propagation is adequate. A general 3-D numerical code for propagation at these temporal scales has been developed by Dr. Potasek, working with post docs A. Paul and S. Kim. (Professors McLaughlin and Shelley participated directly in the early design of the numerical algorithms for this code.)

The code is designed to model propagation along the z axis, including transverse spatial coordinates, and includes higher order dispersion and diffraction terms (e.g. self focusing). It also includes the effects of stimulated Raman scattering and two photon absorption. The code was then used to investigate the relative importance of radial diffraction and material dispersion for a wide range of wavelengths, as well as the significance of various physical parameters – including the laser's temporal and radial pulse width, intensity and wavelength, and the relevant nonlinear and dispersive material properties.

Currently, a more detailed description of the microscopic media, other than phenomologically through interaction coefficients, is being added to the code. These additions are required to model the primary mechanism for optical limiters, reverse saturable absorption.

Bistable Optical Arrays: McLaughlin and Chen have completed an extensive numerical and analytical study of the behavior of arrays of coupled bistable switches, in the presence of diffusion, diffraction, and propagation. Extending the original array work of Firth, diffraction effects on transverse "crosstalk" are focused upon. A coupled system of partial differential equations for three fields (the envelope of the forward propagating electromagnetic wave, the envelope for the backward propagating wave, and a media field), in two space (longitude and transverse) and one time, is studied numerically. The numerical code is carefully benchmarked by comparison with exact solutions and a new asymptotic reduction. The switching and addressing properties for single and multiple pixels are then studied, and estimates of optimal pixel density are made. Performance is found distinctly better for focusing, than for defocusing, nonlinearities. In addition, a new reduced map is derived by an asymptotic averaging process. This map may be viewed as a generalization of a map introduced by Firth, generalized to include some aspects of diffraction through the explicit dependence of the generalized map upon the Fresnel number. The predictions of this map are then compared with full numerical simulations. and they are found to be remarkably accurate. Thus, the map provides an efficient tool for initial investigations of the effects of diffraction on "crosstalk". Also, it should be emphasized that the numerical algorithm (a split step method incorporating characteristics) can be easily generalized to include additional physical effects as they prove to be relevant. This study formed Chen's Ph.D. dissertation.

Precursor Waves: McLaughlin and Ueda's work on "precursor waves" has focused upon the behavior of these waves in idealized models of nonlinear electromagnetic media, and begins with model media which are completely integrable via "soliton mathematics". Precursors, the first wave to impinge into a medium, are potential sources of initial damage which, if severe enough, could render irrelevant damage studies caused by subsequent waves. Thus, precursors could be of fundamental importance in assessing potential sources of laser-eye damage and other laser hardening problems. In addition, precursors represent very general phenomena. For example, the very same equations which are used to describe precursor propagation in laser amplifiers provide idealized models of vast storm tracks in our weather system. Such generic behavior is a natural candidate for theoretical and mathematical investigations.

While precursors are well understood in linear systems, very little is known for nonlinear systems. Even in the integrable soliton models, precursors are the least understood amongst the integrable wave types. For example, while the far field asymptotic behavior is known to be described by certain self-similar solutions called Painleve Transcendents, almost nothing is known about the asymptotic approach to these solutions, about their dependence upon boundary and signaling data, and the generality of self-similar precursors in nonintegrable systems. Such are the issues that Ueda and McLaughlin are addressing.

A numerical study of precursor waves for different classes of nonlinear media has been implemented, a study which focuses upon the signaling boundary value problem. These studies are nearly complete for integrable nonlinearities, with nonintegrable examples remaining. The authors identify precise distinctions between amplifying and attenuating media, and also identify distinctions which originate from different classes of boundary and initial data. For example, in amplifying media "supersonic solitons" constitute the first precursors, but they only arise from extended, noncompact support, data. The approach to the self-similar precursors is also investigated. And, perhaps most importantly, the amount of energy carried by the precursor is quantified and compared with that carried by the remainder of the spatial profile of the wave. Once the numerical experiments are extended to a wider class of nonlinearities (which are not completely integrable), the dependence and sensitivity of the self-similar profiles to the detailed nature of the nonlinearity will be determined.

Precursors and Storm Tracks: The work on model electromagnetic precursors is an outgrowth of a project on precursor waves and storm tracks in the atmosphere, carried out by McLaughlin, in collaboration with Majda, Overman, and Ueda. Recent studies in the Atmospheric literature have presented significant observational and computational evidence for downstream development of mid-latitude storm tracks from regions of enhanced baroclinicity, in contrast to global modal development. These studies make the important suggestion that the precursor waves generating this downstream development might be much more predictable than the details of the trailing storm track. Observational and computational evidence for downstream development exists for both the northern and southern hemispheres. Downstream development is characterized by nonlinear evolution of baroclinic wave packets where the group velocity of the packet exceeds the phase velocity of individual eddies.

A very simple qualitative model has been introduced by Majda, et.al., which captures and clarifies many of the essential features of the process of baroclinic downstream development as described in the above work. While realistic storm tracks are not weakly nonlinear, an existing weakly nonlinear model of Pedlosky for two layer inviscid quasi-geostrophic dynamics can be used to study the leading downstream edge of the baroclinic storm packet. First, within the model transparent analogues of the essential features of the storm are identified, such as unstable packets with group velocities exceeding phase velocities and structures which qualitatively resemble storm track data. Then it is demonstrated that the simplified weakly nonlinear model generates downstream development from a localized region of enhanced baroclinicity in a fashion qualitatively identical to that seen in the prototypical initial value studies of the full two layer dynamics. An exact theory is presented for the leading wave precursor via similarity solutions of the sine-Gordon equation. It is also demonstrated that this downstream development process is very robust and persists through localized random perturbations. Finally, the interaction of downstream development with the global modes of baroclinic instability is studied within the simple model. Roy Goodman is writing his Ph.D. dissertation in this area.

Phase Separation in Polymer-Liquid Crystal Systems: Professor E has been studying polymer/liquid crystal systems, which are the basis for a wide range of new generation smart materials and display devices including micro-filters, switchable windows, polymer-stabilized displays. For example, the Air Force (T. Bunning and R. Sutherland at Wright-Patterson AFB) is interested in using holographic gratings for laser hardening of sensors.

The purpose of the modeling and simulation effort is to understand the dynamics sufficiently well in order to control morphology. This is a very challenging task since the dynamics of a polymer network is not well understood. In addition PDLCs (polymer dispersed liquid crystals) and PSLCs (polymer stabilized liquid crystals) are typically driven by polymerization instead of thermal or mechanical quench, and the polymerization process is far from being understood.

There are two aspects to the problem – the energetics and the dynamics. The energetics of polymer-liquid crystal systems is relatively well understood. In contrast, little is known about the dynamics of such systems. E has, therefore, concentrated his effort on the dynamic aspect. Earlier E and Palffy-Muhoray ["Phase separation of incompressible systems", Phys. Rev. E, Rapid Communications, 55, 1997] showed how to correct the classical Cahn-Hilliard-de Gennes equations by treating properly the incompressibility constraint. The resulting model is non-local and can be regarded as the "level 0" hydrodynamical model. Later E and Felix Otto ["Thermodynamically driven incompressible fluid mixtures", J. Chem. Phys. 107(23), 10177-10184, 1997] showed that the nonlocal model gives rise to qualitatively different behavior in the interfacial regime, and provided the first consistent explanation for the widely observed pinning transition in the phase separation of polymer systems.

Building on this work, E and Otto ["Network formation and visco-elastic effects in polymer phase separation", in preparation] have studied the full visco-elastic dynamics for polymer systems. This is an improvement of the earlier model by Doi and Onuki in that E and Otto took into account the effects of bulk viscosity and bulk modulus which turns out to be crucial in stabilizing the polymer network. The visco-elastic model provided explanation for several important characteristics of polymer phase separation: 1. The time and length scales of the initial instability are much larger than that for phase separation of binary (small molecule) fluid mixtures. In fact the character of the instability is qualitatively different. 2. In a polymer/solvent system. even though polymer concentration may be substantially smaller than that of the solvent, solvent-rich droplets form instead of polymer-rich droplets. 3. The morphology is characterized by the formation of a polymer network which persists for a long time. Shear viscosity destabilizes the polymer network, while bulk viscosity stabilizes it. Although some of this was also independently discovered by Tanaka, et.al. in Japan, the work of E and Otto is much more complete and systematic.

This work represents substantial progress for the understanding of the dynamics of flexible polymer systems. Currently E and his post-doc Muratov are extending this to rigid polymer systems. namely polymer/liquid crystal systems.

Light Propagation in General Homogeneous Media: Professor E (with Palffy-Muhoray and Yuan) has been developing efficient tools for describing the optical response in general homogeneous media, one of the most practical issues for display applications. Many liquid crystal systems contain dichroic dyes and are chiral. Hence one needs to solve Maxwell's equations in cases where the dielectric tensor is biaxial. and the real and imaginary parts do not commute. E, Yuan and Palffy-Muhoray, have developed a formalism which allows them to obtain, for the first time, explicit analytic solutions for the general case. They have implemented this description in a 4×4 matrix propagation method.

Orientational Ratchets in the Photoalignment of Liquid Crystals: E and Palffy-Muhoray have studied photoalignment of liquid crystals in the presence of dichroic dyes. They found that the torque which reorients the liquid crystal does not come from the transfer of angular momentum from the light. but instead it is the result of an "orientational ratchet" mechanism, where the dye plays the role of a Brownian motor. This is interesting both from a theoretical and technological point of view. It makes a connection between the interesting transport issues in biological systems and in liquid crystal systems. It also has important application in display technology (photobuffing of displays), artificial muscle materials (photoelastomers), microactuators and positioners, as well as for optical information storage.

Singularities of Incident or Reflected Waves with Diffracted Waves: Ting (with Keller) has resolved the classical problem of the singularity near the line of tangency of an incident wave or reflected wave and a diffracted wave which was first posed by M.J. Lighthill in 1949, when he studied the singularities of the conical solution of a wing in supersonic flow. The analysis was presented in GaMM97 and will soon be published in ZaMM. The abstract of the paper is:

"In linear theory, a moving weak shock remains tangent to a weak diffracted wave, and the flow is singular at the point of tangency. The path of this point is a singular ray. To correct this defect of linear theory we introduce a canonical nonlinear elliptic problem in the neighborhood of the singular ray. The solution can be computed numerically in a finite domain by matching it to the outer solution. We also formulate a similar canonical problem for the interaction of a weak expansion wave with a weak diffracted wave. These two canonical solutions enable us to resolve the singularities in the self-similar flow fields behind plane shocks diffracted by wedges and corners. and also in the steady flows past thin wings with supersonic leading edges."

Vortex Filaments, Solid Boundaries, and Sound Generation: Ting has continued his investigations of vortex filaments, solid boundary interaction and sound generation, which were supported by an earlier AFOSR grant. A numerical code was developed which implements (1) the theoretical analysis of Knio and Ting ["Vortical Flow outside a Sphere and Sound Generation", SIAM J. Appl Math, Vol 57, 972-981, 1995] for the interaction of a vorticity field, a solid sphere and the far field sound and (2) the vortex filament method supplemented by the asymptotic theory [Klein, Knio, Ting, "Representation of Core Dynamics in Slender Vortex Filament Simulations", Phys. Fluids A., Vol. 8, 2415-2425, 1996] for the motion of a slender filament. The code is employed to compute the far field sound generated by the passage of a slender ring over a rigid sphere. Both coaxial and noncoaxial passage events are analyzed in the computations, as well as the effects of initial core size and asymmetric perturbations. The results are presented by Knio, Ting and Klein ["Interaction of a slender vortex filament with a rigid sphere: dynamics and far-field noise", J. Acous. Soc Amer., Vol 103, 83-98, 1998.] An invited lecture was presented by Ting in the 1997 IUTAM Symposium on Dynamics of Slender Vortices. The title is "Asymptotic theory of slender vortex filaments - old and new" with R. Klein and O. Knio as co-authors. The abstract is:

"We give a brief review of the asymptotic theory of slender vortex filaments to emphasize i) the choices of scalings, small parameters and the distinguished limit, ii) the consistency conditions, iii) the optimum and similar and non-similar viscous vortical core structures and iv) their applications to complement experimental investigations. We present highlights of several extensions of the asymptotic theory: the analyses for core structures with axial variation. for the interaction of filaments with a solid body and sound generation, and for a filament in a background rotational flow. We then outline the vortical flow problems currently under investigation."

In the same symposium, an invited talk was presented by Knio with Ting as the co-author. The title is "Noise Emission due to Slender Vortex/Solid Body Interactions – Ring/Sphere Interaction at High Reynolds Number", and the abstract is:

"Interactions of a slender vortex ring with a stationary rigid sphere are analyzed using a 3D, Lagrangian vortex element scheme which discretizes and tracks the filament centerline using smoothed vortex elements. The filament self-induced velocity is obtained from a desingularized Biot-Savart law that reflects the correct asymptotic behavior of the core vorticity distribution. The effect of the sphere is represented in terms of a potential velocity field that is expressed as a line integral along the image of the filament centerline with regular weight functions. It is shown that the acoustic emission due to the interaction between the filament and the sphere essentially consists of dipoles and quadrupoles whose strengths and orientations are determined by the time-evolution of the weighted first and second moments of vorticity, respectively. The scheme is used to analyze the sound generated during the passage of slender vortex rings near a solid sphere."

3 Personnel Associated with Research Effort

- FACULTY

  David W. McLaughlin:
  Weinan E:
  Lu Ting

  In addition, while not receiving explicit support from this grant, Professors Jalal Shatah and Michael Shelley were continually involved in parts of the research effort.

- POST-DOCS

  Felix Otto;
  Tetsuji Ueda;
  Cyril Muratov;
  Andrew Paul;
  Sukkeun Kim

In addition, while not receiving explicit support from the grant, David Cai and Jim Wielaard have been involved in parts of the research effort.

- GRADUATE STUDENTS

  Mark Winograd;
  Yuchi Chen;
  Roy Goodman

- OTHER

  Jared Bronski (Ex PhD Student, now Asst. Prof., Stanford);
  Analisa Calini (Asst. Prof., Charleston College);
  Constance Schober (Ex PhD Student, Asst. Prof., Old Dominion);

  Thomas Ernaux (Prof., Universite Libre de Bruxelles);
  Philip Rosenau (Prof, Tel Aviv University);

  Ruth Pachter (Research Scientist, Wright Patterson AFB);
  Mary Potasek (Research Scientist, Brooks Air Force Base)

Publications

Submitted and in Preparation

Journals

1. D. Cai, D.W. McLaughlin, J. Shatah, "Spatial Temporal Chaos and Effective Stochastic Dynamics for a Near Integrable Non-Linear System, submitted Phys. Rev. Lett (1998);
2. Yuchi Chen and D.W. McLaughlin, "Focusing-Defocusing Effects for Diffusion Dominated Bistable Optical Arrays", to be submitted to J. Opt. Soc. America (Sept, 1998);
3. D.W. McLaughlin, T. Ueda, "Precursors for Nonlinear Wave Equations", in preparation;
4. A. Majda, D.W. McLaughlin, E. Overman, T. Ueda, "A Weakly Nonlinear Model of Storm Tracks", to be submitted, Fall (1998);

5. A. Calini, D.W. McLaughlin, J. Shatah, C. Schober, "Homoclinic Chaos in PDE's Without Even Symmetry", in preparation;

6. Ting, L. and Keller, J.B., "Weak Shock Diffraction and Singular Rays", an extended abstract to appear in ZaMM, in preparation,;

7. Ting, L., Klein, R. and Knio, O., "Asymptotic Theory of Slender Vortex Filaments - Old and New", to be published in the Proceedings of the IUTAM Symposium on Dynamics of Slender Vortices, RWTH Aachen, August 31-September 3, 1997, Kluwer Academic Publ., Dordrecht;

8. Knio O. and Ting, L., "Noise Emission due to Slender Vortices - Solid Body Interaction", to be published in the Proceedings of the IUTAM Symposium on Dynamics of Slender Vortices, RWTH Aachen, September 1-3, 1997, Kluwer Publ., Dordrecht;

9. Weinan E. and P. Palffy-Muhoray, "Dynamics of Filaments During the Smectic-Isotropic Transition", submitted to J. Nonlinear Sciences;

10. Weinan E and F. Otto, "Network formation and visco-elastic effects in polymer phase separation", in preparation;

11. Haijun Yuan, Weinan E, Tamas Kosa and P. Palffy-Muhoray, "Analytic 4 times 4 propagation matrices for homogeneous media", To appear in SID 98 DIGEST;

12. Weinan E, Tamas Kosa and P. Palffy-Muhoray, "Orientational ratcheting in the photoalignment of liquid crystals", preprint;

13. P. Palffy-Muhoray and Weinan E, "Orientational ratchets and angular momentum balance in the Janossy effect", to appear in Mol. Crystal and Liquid Crystals;

14. V. Krylov and P. Rosenau, "Solitary Waves in an Elastic String", Preprint, Tel Aviv University;

15. D. Muraki, M. Shelley, and T. Ueda, "Laser Beam Undulations in Nematic Liquid Crystals: Polarization Effects", Preprint, Courant Institute.

Accepted

Journals

1. Shan Jin. D. Levermore and D. McLaughlin, "Semiclassical Limit of the Integrable, Defocusing NLS Equation", accepted Comm. Pure Appl. Math (1998):

2. A. Majda, D.W. McLaughlin and E. Tabak, "A One Dimensional Model for Dispersive Wave Turbulence", J. Nonlinear Science, 7, 9-44 (1997);

3. Y. Li and D.W. McLaughlin, "Homoclinic Orbits in Discretized Perturbed NLS Systems", J. Nonlinear Science, 7, 211-269, (1997);

4. D.W. McLaughlin and J. Shatah, "Homoclinic Orbits via Lyapanov-Schmidt Methods", Proc. Conf. in Honor of P. Lax and L. Nirenberg, Venice AMS (1997);

5. J.C. Bronski, D.W. McLaughlin and M. Shelley, "On the Stability of Time-Harmonic Localized States in a Disordered Nonlinear Medium", J. Stat. Phys. (1997);

6. Y. Li. D.W. McLaughlin. J. Shatah and S. Wiggins, "Persistent Homoclinic Orbits for a Perturbed Nonlinear Schroedinger Equation", Comm. Pure Appl Math., 49. 1175-1255, (1996);

7. D.W. McLaughlin, D. Muraki and M. Shelley, "Self-focussed Optical Structures in a Nematic Liquid Crystal", Physica D, 97, 471-497 (1996);

8. D.W. McLaughlin, E. Overman, S. Wiggins and C. Xiong, "Homoclinic Orbits in a Four Dimensional Model of a Perturbed NLS Equation: A Geometric Singular Perturbation Study", Dynamics Reported, 5, 190-286 (1996);

9. D.W. McLaughlin and J. Shatah. "Homoclinic Orbits for PDE's in Recent Advances in Partial Differential Equations", eds. R. Spigler, S. Venakides - Proc Conf. in honor of P. Lax and L. Nirenberg, Venice - AMS 1996, 281-299;

10. D.W. McLaughlin and E. Overman, "Whiskered Tori for Integrable pde's" Chaotic Behavior in Near Integrable pde's", Surveys in Appl Math 1, 83-203, (1995);

11. D.W. McLaughlin, D. Muraki, M. Shelley and X. Wang, "A Paraxial Model for Optical Self-Focusing Nematic Liquid Crystal", Physica D, 26. 55-81 (1995);

12. D.W. McLaughlin. "Whiskered Tori and Chaotic Behavior in Nonlinear Waves", Proceedings of International Congress of Mathematicians, Zurich, 1494-1493 (1995);

13. A. Calini, N.M. Ercolani, D.W. McLaughlin, and C.M. Schober, "Mel'nikov Analysis of Numerically Induced Chaos in Nonlinear Schrodinger Equation", Physica D, 89, 227-260, (1995);

14. Y. Chen. "Diffraction Effects on Diffusive Optical Bistability and Optical Memory", PhD Dissertation, Princeton University (1997);

15. Knio, O., Klein, R. and Ting, L., "Interaction of a Slender Vortex Filament with a Rigid Sphere: Dynamics and Far-Field Noise", J. Acoust Soc. Amer., 103-83-98, (1998);

16. Miksis, M.J. and Ting, L., "Structural Acoustic Interactions and On Surface Conditions". IMA Volumes in Mathematics and its Applications. 86. "Computational Wave Propagation", Editors, B. Engquist and G.A. Kriegsmann. 165-178, Springer-Verlag, (1997);

17. Knio, O.M. and Ting. L.. "Vortical Flow Outside a Sphere and Sound Generation", SIAM J. Appl. Math, Vol. 57, No. 4, 972-981, (1995);

18. Bauer, F., Maestrello, L. and Ting, L., "Acoustic Wave Propagation in Unsteady Moving Media", J. Acoust. Soc. Amer., 99, 1291-1305, (March 1996);

19. Klein, L., Knio, O. and Ting L., "Representation of Core Dynamics in Slender Vortex Filament Simulations", Phys. Fluids A., 8, 2415-2425, (1996);

20. Ting, L., "On Surface Conditions for Structural Acoustic Interaction in Moving Media", SIAM J. Appl. Math., 55 369-389, (1995);

21. Miksis. M.J. and Ting. L.. "Panel Oscillations and Acoustic Waves", Appl. Math. Letters. Vol. 8. 1, 37-42, (1995);

22. Keller, J.B., King, A. and Ting, L., "Blob Formation", Phys Fluids, 7, 226-228, (1995);

23. Klein, R. and Ting, L.. "Theoretical and Experimental Studies of Slender Vortex Filaments". Appl. Math. Letters, 8, No. 2, 45-50, (1995);

24. Weinan E. and P. Palffy-Muhoray, "Phase Separation of Incompressible Systems", Phys. Rev. E, Rapid Communications, (April 1997);

25. Weinan E and P. Palffy-Muhoray, "Wavelength Selection in Slowly Quenched Systems", Molecular Crystal and Liquid Crystals, 292-345 (1997);

26. Weinan E and F. Otto, "Thermodynamically driven incompressible fluid mixtures", J. Chem. Phys. 107 (23), 10177-10184, (1997);

27. Weinan E, "Nonlinear continuum theory of smectic A liquid crystals", Arch. Rat. Mech. Anal., 137 (1997) 159-175;

28. T. Ueda and P. Hagan, "Evolution of Bloch electrons with Applied Electromagnetic Fields: the Semiclassical Equations", European Journal of Applied Mathematics (1996);

29. A.E. Paul. J.A. Bolger. A.L. Smirl. J.G. Pellegrino, "Time Resolved Measurements of the Polarization State of Four-Wave Mixing Signals from GaAs Multiple Quantum Wells," J. Opt. Soc. Am B 13, 1016, (1996);

30. M.J. Potasek, "Interplay of Dispersive and Diffractive Effects of Ultrashort Laser Pulses in Nonlinear Ocular Media", Laser-Tissue Interaction VII, 2681, 411-419 (1996);

31. M.J. Potasek and A.E. Paul, "Investigation of Nonlinear Ocular Media using Femtosecond Laser Pulses", Laser and Noncoherent Ocular Effects: Epidemiology, Prevention, and Treatment, 2974, 66-74 (1997).

Conferences

1. D. McLaughlin, "Whiskered tori and chaotic behavior in nonlinear waves". Internal Congress of Mathematicians, Zurich, 1994;

2. D. McLaughlin, U. of New Mexico/CNLS Distinguished Lecturer, 1995;

3. D. McLaughlin, Wayne State, Distinguished Lecturer, 1995;

4. D. McLaughlin. INRIA/SIAM Invited Lecturer, Rocquencourt, France, 1995:
5. D. McLaughlin, AMS/SMM Invited Address, University of Guanajuato, Mexico, 1995;
6. D. McLaughlin, "Random Behavior in PDE", Invited Lecturer, IX David Alcaraz Spinola Conference, Univ. Mexico, 1995;
7. D. McLaughlin, "Homoclinic Orbits via Lyapanov-Schmidt Methods," Invited Address, Conference for P. Lax and L. Nirenberg, Venice, Italy, 1996;
8. D. McLaughlin, "Precursors for Baroclinic Storm Tracks", Invited Lecture, 17th Annual International Conference, CNLS, Los Alamos (May 1997);
9. D. McLaughlin, "Homoclinic and Chaotic Behavior in Nonlinear PDE's" Invited Lecture, IMA Conference on Dynamics (Oct 1997);
10. D. McLaughlin and Jalal Shatah, "Geometric Singular Perturbation Theory and Homoclinic Behavior in PDE's", Invited Short Course, AMS-Mexico Meeting (Dec 1997);
11. L. Ting and J.B. Keller, "Weak Shock Diffraction and Singular Rays", Invited Lecture, GAMM, Germany (March 1997);
12. L. Ting, R. Klein, and O. Knio, "Asymptotic Theory of Slender Vortex Filaments – Old and New", Invited Lecture, IUTAM Symposium on Dynamics of Slender Vortices, Germany (Sept 1997);
13. Weinan E, "Invariant measures for stochastic Burgers equation", Invited Lecture, Conferences in PDEs, Northwestern University, (March, 1998);
14. Weinan E, "Phase separation in polymer systems", Invited Lecture, Workshop on Nonequilibrium Dynamics, Los Alamos, (April, 1998);
15. Weinan E, "Exact results on PDEs for Burgers equation", Invited Lecture, Turbulence in the 21 Century, Los Alamos, (May, 1998);
16. Weinan E, "Pattern formation in liquid crystal phase transition", Invited Lecture, IMA Workshop on Continuum Mechanics and Nonlinear PDEs, (June, 1998);

17. Weinan E and P. Palffy-Muhoray, "Dynamics of Filaments During Smectic Isotropic Phase Transition", Workshop on Applied Mathematics, Penn State University, (October 1996);

18. Weinan E and B. Engquist, "Unsteady Prandtl's Equation", Invited Lecture. Conference on Shock Wave Theory, Atlanta, GA, (June 1997);

19. Weinan E, "Deterministic and Stochastic Aubry-Mather Theory from a PDE Point of View", Invited Lecture, First Tian-Yuan Conference in Mathematics, Berkeley, (June 1997);

20. M.J. Potasek and A.E. Paul, "Ultrafast Pulse Propagation in Nonlinear Ocular Media", Invited Lecture, 4th AFOSR Ultrashort Laser Collaborative Workshop, San Antonio, TX (March 1996);

21. M.J. Potasek, "Infrared Models of Heating and Damage: Ultrashort Laser Eye Damage". Invited Lecture, Infrared Lasers and Millimeter Waves Workshop: The Links Between Microwaves and Laser Optics, San Antonio, TX (January, 1997);

22. M. Shelley and T. Ueda. "The Nonlocal Dynamics of Stretching, Buckling Filaments", AMS-SIAM Conference Proceedings (1996);

23. A.L. Smirl, A.E. Paul, J.A. Bolger, "Ultrafast Time-Resolved Ellipsometry of Four-Wave Mixing Signals: Measurement of Coherent Nonlinear Processes in Semiconductors", Nonlinear Optics: Materials, Fundamentals, and Applications '96, paper NWA2-1, Maui, Hawaii, (July 8-12, 1996);

24. A.L. Smirl, J.A. Bolger, A.E. Paul, J.G. Pellegrino, "Time Resolved Measurements of the Polarization State of Coherent Four-Wave Mixing Signal from GaAs Multiple Quantum Wells", Quantum Electronics Laser Science Conference QELS '96, paper QWB2, Anaheim, CA., (June 2-7, 1996).

Interactions/Transitions

Participation/Presentations At Meetings, Conferences, Seminars, Etc

E: Co-Organizer: Minisymposium in Dynamics and Defects in Polymer-Liquid Crystal Systems, SIAM Meeting on Material Sciences, Philadelphia,

(May 1997):
E: Co-Organizer: CIMS/ALCOM/NIST workshop on phase separation polymer-liquid crystal systems, April 25-26, New York.

Consultative and Advisory Functions to Other Laboratories and Agencies

McLaughlin:

Advisory Function, Brooks AFB, Mary Potasek, "Advice of short pulse numerical code";

Cooperation, Wright Patterson and Brooks AFB, Ruth Pachter and Mary Potasek, "Initiating Project on light-matter interactions";

Member. Science Policy Committee SIAM;

Member. IMA Board of Governors;

Member. Study Panels for National Science Board and NSF;

Member. Board of Directors, FENOMEC (Univ of Mexico);

Member. Review Panel, Mathematics in Canada;

Member. Site Visit Team, Institute for Advanced Study, NSF;

Member. Advisory Committee Research Strategy Planning of Mathematical and Computer Science Division of Army Research Office;

E:

Member, Review Panel, NSF Proposals in Applied Mathematics

Transitions

McLaughlin's collaboration with members of three laboratories (Courant Institute, Brooks AFB, Wright Patterson AFB) is in its initial stages.

Ting's co-workers: Prof. J.B. Keller, Stanford University: Maestrello, NASA Langley Research Center; Prof. R. Klein, Bergische Universitat, Wuppertal, Germany: and Prof. O. Knio. the John Hopkins University.

Ting: is consultant to ICASE, NASA Langley Research Center Maestrello, and a visitor to the Aerodynamisches Institut, RWTH Aachen working with Prof. E. Krause, the Director, supported by the Alexander von Humboldt Foundation, Bonn.

New Discoveries, Investions or Patent Disclosures

Honors/Awards

McLaughlin

1. 1994: Director, Courant Institute of Mathematical Sciences, New York University;
2. 1994: International Congress of Mathematicians, Zurich, Invited 45 Min Address;
3. 1995: Recipient of 50th Anniversary Medal, University of Mexico;

E:

1993: Alfred P. Sloan Fellowship;
1996: Presidential Faculty Fellowship (NSF);

Ting:

1996: Humboldt Research Award for Senior U.S. Scientists.